Centro de Investigación en Matemáticas

Workshop

Random Trees: Structure, Self-Similarity and Dynamics

April 23-28, 2018

Abstracts

Tom Alberts, University of Utah

Monday, 2:30 – 3:30 PM

Multifractality of Multiplicative Cascades, a Diffusive Point of View

Two of my favorite papers of Ed's study the multifractality of the multiplicative cascade measures on trees, i.e. a quantification of the size of points around which the random cascade measure has a pre-specified local scaling behavior. I'll discuss how ideas from these papers influence an in-progress multifractal analysis of a random measure coming from the Circular-Beta ensemble of random matrix theory (joint with Balint Virag and Raoul Normand), and an earlier attempt of myself and Ben Rifkind to rederive multifractality results for multiplicative cascades from a stochastic calculus point of view.

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Octavio Arizmendi, CIMAT

Monday, 4:00 – 5:00 PM

Spectral distributions of graphs products

In this talk I will survey on results about graphs products and their distributions. We will focus on the free product, which produces tree-like structures and describe its relation with random graphs.

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Maxim Arnold, University of Texas at Dallas

Tuesday, 4:00 – 5:00 PM

Trees for exploration and sensing [tutorial]

In this mostly tutorial talk I will discuss two random tree constructions coming from robotic applications and designed for the motion planning and exploration algorithms. I will describe dynamic arising from these algorithms.

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Krishna Athreya, Iowa State University

Tuesday, 11:30 AM – 12:30 PM

Convergence of the empirical distribution of the income at generation n in a GWB process

Consider a GWB branching process of one type with no extinction. Assume at generation n each person of the nth generation inherits some money from the parent and makes money in his life time subject to some noise. Assume that in each line of descent the income is a discounted random AR series. We show that under some regularity assumptions the empirical distribution function of the income at generation n converges as n goes to infinity in sup norm, i.e. a Glivenko-Cantelli type theorem holds.

Bruno Barbosa, Oregon State University [student speaker]
Thursday, 3:00 – 3:30 PM

Self-Similarity in Level Set Trees of Geometric Random Walks

Level set trees provide an insight into the topology of a function's relative extrema. We apply this idea to a geometric random walk whose displacement kernel is a mix of geometric distributions. The resulting trees are not binary, so we calculate how the parameters of the transition kernel evolve under the operation of Horton pruning. Under the assumption of a mean zero walk, we prove that these trees satisfy asymptotic Horton and Tokunaga self-similarities. Furthermore, we show that for a mean zero geometric random walk, the level set tree is a critical Galton-Watson tree, and iterative pruning leads to a critical binary Galton-Watson tree as predicted in Burd et al. (2000). This is a joint work with C. Campregher, J. Johnson, Y. Kovchegov, and I. Zaliapin.

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Mine Çağlar, Koç University
Monday 9:00 – 10:00 AM

Branching Brownian motion in a Poissonian trap field and skeleton decomposition

We study a branching Brownian motion Z in R^d, among obstacles scattered according to a Poisson random measure with a radially decaying intensity. The branching Brownian motion can be represented as a marked tree, where the marks are the lifetimes and the trajectories. Obstacles are balls with constant radius and each one works as a trap for the whole motion when hit by a particle. Considering a general offspring distribution, we derive the decay rate of the annealed probability that none of the particles of Z hits a trap, asymptotically in time t. We provide an appropriate skeleton decomposition for the underlying Galton-Watson process when supercritical. It is a multi-type branching process, which can also be analyzed in the framework of marked trees. We show through a non-trivial comparison that the doomed particles do not contribute to the asymptotic decay rate. This is joint work with Mehmet Oz and Janos Englander, who have additionally studied optimal survival strategies concerning the population size and the range of Z, as well as the size and position of clearings in R^d. In this lecture, we elaborate on the tree representation as an alternative approach and for possibly generating new research questions.

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Evgenia Chunikhina, Oregon State University [student speaker]
Thursday, 3:30 – 4:00 PM

Entropy of Self-Similar Random Trees

In this talk we examine planted binary plane trees. First, we provide an exact formula for the number of planted binary trees with given Horton-Strahler orders. Then, using the notion of entropy, we examine the structural complexity of random planted binary trees with N vertices. Finally, we quantify the complexity of the tree's structural properties as tree grows in size, by evaluating the entropy rate for planted binary plane trees with N vertices and for planted binary plane trees that satisfy Horton Law with Horton exponent R.

Radu Dascaliuc, Oregon State University
Wednesday, 8:45 – 9:45 AM

Stochastic explosion and lack of uniqueness in non-linear Partial Differential Equations: the case study of the α-Riccati equation [tutorial]

In 1997 Le Jan and Sznitmann introduced a stochastic cascade approach to studying global well-posedness of the 3D Navier-Stokes Equations (NSE). The key ingredient in their existence and uniqueness result was the non-explosive nature of the cascade structure they have considered. In fact, a close connection seems to exist between the natural scaling of the NSE and the explosion property of the associated cascade, which in turn may lead to a possible pathway towards a non-uniqueness for the NSE Cauchy problem. We will explore this connection between stochastic explosion and uniqueness on a much simpler test case: the α-Riccati equation. This differential equation features a non-local quadratic nonlinearity coupled with a dissipative linear term and it can be viewed as a toy problem for the NSE-like systems. In the case $\alpha<1$, we will see how lack of explosion of the associated cascade leads to global well-posedness. In contrast, in the case $\alpha>1$ the cascade is exploding, and we are able to establish both non-uniqueness and finite-time blow up for large initial data. In particular we will discover how stochastic cascade explosion leads to the lack of uniqueness of global solutions for arbitrary small initial data. This research is a joint work with E. Thomann and E. Waymire and is a part of a larger program aimed of bringing together functional-analytic and stochastic techniques to understand qualitative properties of deterministic non-linear PDE.

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Wilfrid Kendall, University of Warwick
Monday, 10:30 – 11:30 AM

Traffic in a random Poissonian network: limit theorems assisted by improper Poisson line processes

A random spatial network in a large disk can be constructed using a stationary and isotropic Poisson line process of unit intensity, with a suitable "cross-country" convention to connect points not lying on the network [2]. Suppose traffic is generated uniformly along "near-geodesics" (connecting pairs of points using a certain lazy algorithm). If the Poisson line pattern is conditioned to contain a line through the centre, then one can show that a scaled version of the traffic flow has asymptotic distribution given by the 4-volume of a region in 4-space, constructed using an improper anisotropic Poisson line process in an infinite planar strip. (Improper Poisson line processes are pervasive in this area: see also the "Poisson line SIRSN" [4,5]) A more amenable representation can be obtained in terms of two “seminal curves” defined by the improper Poisson line process, thus producing a framework for effective simulation from this distribution up to an L^1 error which tends to zero with increasing computational effort [3]. Recently, in his PhD Thesis [1], Gameros has extended these results and discusses a curious connection with mid-20th century British rail traffic.

References:

1. Gameros, R. (2017). Mean traffic behaviour in Poissonian cities. *PhD Thesis*, University of Warwick.
2. WSK. Geodesics and flows in a Poissonian city. *Ann. Appl. Prob.* 21 (2011), no. 3, 801-842.
3. WSK. Return to the Poissonian City. *J. Appl. Prob.* (2014), 15A, 297-309.
4. WSK. From Random Lines to Metric Spaces. *Ann. Prob.* 45 (2017), no. 1, 469–517.
5. Kahn, J. Improper Poisson line process as SIRSN in any dimension. *Ann. Prob.* 44 (2016) no. 4, 2694–2725.

Yevgeniy Kovchegov, Oregon State University
Thursday, 10:30 – 11:30 AM

Random Trees and Their Applications: Metric Trees

This is a second part of the talk focused on self-similarity for trees (see Ilya Zaliapin for the first part). We consider trees with edge lengths (hence every tree is a metric space). We suggest a general definition of self-similarity via the operation of generalized tree pruning. The latter encompasses a number of previously studied discrete and continuous pruning operations, including the tree erasure studied by Jacques Neveu and Horton pruning studied by Ed Waymire. The proposed framework unifies the self-similarity results of Waymire and that of Neveu for the critical binary Galton-Watson trees; it also suggests multiple other types of pruning that preserve the critical binary Galton-Watson distribution with edge lengths. We introduce a hierarchical branching process that generates a rich class of self-similar trees. We emphasize a natural appearance of a one-parametric family of critical Tokunaga trees with edge lengths that preserve many of the symmetries of the critical binary Galton-Watson trees with edge lengths and reproduce the latter at a particular parameter value.

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Peter Otto, Willamette University
Monday 11:30 AM – 12:30 PM

Coalescent processes and mean minimal spanning trees of irregular graphs

It is known that in many cases the cluster dynamics of a random graph process can be replicated with the corresponding coalescent process. The cluster dynamics of a coalescent process (without merger history) is reflected in a auxiliary process called the Marcus-Lushnikov process. The merger dynamics of the Marcus-Lushnikov process will correspond to a greedy algorithm for finding the minimal spanning tree in the respective random graph process. This observation allows one to express the limiting mean length of a minimal spanning tree in terms of the solutions of the Smoluchowski coagulation equations that represent the hydrodynamic limit of the Marcus-Lushnikov process corresponding to the random graph process. In this talk, I will present joint work with Y. Kovchegov and A. Yambartsev on breaching the gap between the Smoluchowski coagulation equations for Marcus-Lushnikov processes and the theory of random graphs, concentrating on the case of irregular bipartite graphs and deriving the limiting mean length of minimal spanning trees with random edge lengths for this sequence of graphs.

Julia Palacios, Stanford University
Tuesday, 10:30 – 11:30 AM

State-space exploration of Tajima's Trees

Estimation of population genetic parameters from ancient and present-day samples of genomic data is a core goal of population biology. Modeling the sample's ancestry and the mutation processes of completely linked genomic segments allows such inference. Coalescent-based methods rely on the Kingman-coalescent genealogy to model the sample's ancestry. Unfortunately, the state space of genealogies grows superexponentially with the number of samples and hence, inference is computationally challenging for large number of samples. Here, we present a new Bayesian model that relies on a lower resolution coalescent process called Tajima-coalescent. A Tajima's genealogy includes coalescent times and a ranked tree shape, and has a drastically smaller state space than that of a coalescent genealogy which includes coalescent times and a labeled topology. We provide a new algorithm for likelihood calculation under the infinite sites model for mutation and provide an efficient algorithm for exploring the reduced latent state space. We compare the performance of our algorithm and lower resolution latent model with state-of-the-art algorithms in population genetics.

Enrique Thomann, Oregon State University
Wednesday, 11:00 AM – 12:00 PM

Self-Similar branching processes and the Navier-Stokes equations – Remarks on explosion and symmetry breaking

Self-similar solutions of the Navier Stokes equations (NSE) have been considered as possible scenarios for the formation of singularities in the solutions of these equations. Branching processes were introduced by Le Jan and Sznitman to represent solutions of the NSE as expected values of a multiplicative functional. When self-similar solutions are considered, the resulting branching process inherits this characteristic. The talk, based on joint work with Radu Dascaliuc, Nick Michalowski and Ed Waymire, will focus on properties of the self -similar branching process associated with the NSE and possible avenues to explore symmetry breaking in the solutions and their uniqueness properties.

Scott D. Peckham, University of Colorado, Boulder
Friday, 11:30 AM – 12:30 PM

Results for a Class of Rooted, Binary, Random Trees with Fixed Height

Many random and deterministic tree graph models have been introduced and analyzed over the years. Much of this work was driven by a desire to understand a curious regularity that is observed in the topological structure of river networks, known as the Horton-Strahler law of stream numbers. Shreve's random topology model received a lot of attention and leads to the result that the overwhelming majority of binary tree graphs with a fixed number of leaves, n, have a Horton-Strahler bifurcation ratio of 4 as n goes to infinity. However, modern analysis of data for large river networks shows that they typically have bifurcation ratios that are significantly different than 4. This disagreement with the random topology model is striking and implies that river networks cannot be viewed as being chosen randomly from the set of all rooted, binary trees with a fixed number of leaves, and in fact are being chosen from a much "smaller" class of binary trees. This has led to the pursuit of new random tree models that are able to produce any bifurcation ratio in the allowed range, i.e. values greater than or equal to 2. The main purpose of this talk is to introduce a new random tree model for rooted, binary trees that is tractable and that has this flexibility. Each random tree has the same height, H, and the model has a single parameter, p, in (0,1]. Letting $n_1(p)$ and $n_2(p)$ be the random number of first and second order Horton-Strahler streams, the limit of $E[n_1(p)]/E[n_2(p)]$ as H goes to infinity (viewed as the bifurcation ratio) can be computed. Varying p from 1 down to 0 produces bifurcation ratios between 2 and infinity. These are preliminary results, and there are indications that this model can yield many additional insights. In addition to these results, an analogy between normal numbers and trees from a variant of Shreve's model will be discussed.

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Gabriel Perez, University of Iowa [student poster presentation]
Thursday, 3:30 – 4:00 PM

Scaling of Peak Flows with Variable Flow Velocity in Random Self-Similar Networks

We study the effect of spatial and temporal variability of flow velocity on the scaling statistics of peak flows and time to peaks in Random Self-similar Networks (RSNs). This work is a follow up on a published work where scaling of peak flows on RSNs were examined under the assumption of constant flow velocity. We analyze the scaling exponents associated to four properties of the flow hydrograph $h_\omega(t)$ for ensembles of RSNs with respect to Horton order ω, these are i) the expected value of peak flows $E[max_t h_\omega(t)]$, ii) the peak of the mean hydrograph $max_t E[h_\omega(t)]$, iii) the expected time to peak flow $E[T(max_t h_\omega(t))]$, and iv) the time to peak flow of the mean hydrograph $T(max_t E[h_\omega(t)])$. We start with a generic functional form to characterize variations of at-a-station mean velocity with respect to flow depth d and channel slope γ given by $v = f_r d^m \gamma^n$ (where f_r is a constant friction factor, and m and n are constant exponents), which is consistent with Manning's, Chezy and Darcy-Weisbach equations under appropriate assumptions on channel geometry. Then we assume that hydraulic geometric (HG) channel features (width, slope, friction) obey statistical simple scaling (SSS) with respect to Horton order. These two assumptions allow us to collapse the at-a-station and downstream variations of flow velocity into the form $v(t) = v_o q(t)^{\lambda_1} A^{\lambda_2}$ where $q(t)$ is the stream discharge, v_o is a random variable, λ_1 depends on m and at-a-station HG and λ_2 depends on m, n and downstream HG. We develop mathematical expressions to relate the scaling the exponents of hydrograph properties to λ_1, λ_2 and river network geomorphic features. These mathematical expressions serve as diagnostic and predictive tools for floods. The mathematical expressions developed here allow us to conclude that the scaling exponent of peak flows is independent of runoff intensity, which does not support previous results in idealized networks. These discrepancies are explained. Joint work with Ricardo Mantilla.

Jorge M. Ramirez, Universidad Nacional de Colombia, Sede Medellin
Thursday, 11:30 AM – 12:20 PM

Statistical scaling for the invariant distribution of discharge in self-similar river networks

We consider the linearized mass and momentum conservation equations in a self-similar river network that responds to random but instantaneous precipitation events. Under a stationary precipitation regime, the discharge is an ergodic piecewise-deterministic Markov process. The invariant distribution of such process can be characterized explicitly in terms of: the river network's geometry, the properties of the precipitation process, and the basin's residence times. We use this characterization to show that under this model, statistical scaling of the discharge occurs in the limit of spatially homogeneous river networks with hillslope residence times much larger than those of the streams. In this talk we will heavily draw from Ed Waymire's contributions to basin-scale mathematical models of channel routing, self-similar random river networks and statistical scaling.

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Krishnamurthi Ravishankar, NYU-Shanghai
Friday, 10:30 – 11:30 AM

Continuum limit of voter model perturbation and Brownian net with killing

This talk will focus on particular class of one dimensional voter model perturbations which involve bulk and boundary nucleation of colors. The color genealogy (in the discrete setting) is based on a study of coalescing branching random walks with killing going backwards in time. The killings correspond to bulk nucleations while the branchings keep track of potential boundary nucleations. In order to determine the actual boundary nucleations and the colors at any time the process is run forward in time starting from an initial set of colors. In the continuum limit the backward object is the Brownian net with killing and the color assignment uses properties of special points of the Brownian net. This talk is based on joint work with C.M. Newman and E. Schertzer.

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Deena Schmidt, University of Nevada Reno
Tuesday, 9:00 – 10:00 AM

Complexity reduction for stochastic network models in biology

Markov processes are widely used to model the dynamics of biological processes evolving on networks. Complexity reduction aims to capture the essential dynamics of the process via a simpler representation, with minimal loss of accuracy. The stochastic shielding approximation is a novel dimension reduction method that has been used to simplify stochastic network models arising in neuroscience, such as randomly gated ion channel models, but applies broadly to many biological systems. In this talk, I will describe the stochastic shielding approximation and explore the robustness of the method under conditions of timescale separation and population sparsity.

Sunder Sethuraman, University of Arizona
Tuesday, 2:30 – 3:30 PM

On the growth of nonlinear preferential attachment random trees

We consider an evolving preferential attachment random tree model where at discrete times a new node is attached to an old node, selected with probability proportional to a weight function of its degree. Different behaviors emerge depending on whether the weight function is linear, sublinear or superlinear. In the linear and sublinear cases, the empirical degree distributions are tight, and converge to nontrivial distributions on N. However, in the superlinear situation, the graph evolution condenses, that is a.s. in the limit graph there will be a single random node with infinite degree, while all others have finite degree. In this talk, we discuss this phenomena and related results which shed light on the general picture.

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Sara M. Vallejo-Bernal, Universidad Nacional de Colombia, Sede Medellín, School of Mathematics [student poster presentation]
Thursday, 3:30 – 4:00 PM

Validation of a Conceptual Stochastic Rainfall-Runoff Model Applied to Tropical Watersheds

The transformation of rainfall into runoff in a river basin is, perhaps, one of the most important phenomena of the hydrological cycle because it allows forecasting how the discharge responds to a precipitation event. Our aim here is to validate a conceptual stochastic linear model for this phenomenon in a watershed under a stationary precipitation regime. For this, we test the hypothesis that the precipitation over the river basin follows a Compound Poisson Process and evaluate the existence and properties of the invariant distribution of the discharge, using the precipitation data at hourly resolution of around 300 raingauges, and the discharge data at daily resolution of around 600 gauging stations over the Colombian territory obtained from the hydrometeorological measurement network of IDEAM. We do not pretend to simulate or predict water flows on real catchments, but rather to mathematical understand the underlying relations. Joint work with Jorge Ramirez.

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Ed Waymire, Oregon State University
Friday, 9:00 – 10:00 AM

Strange Bedfellows: Burgers, Yule, and Poisson

The Lejan-Sznitman cascade applied to the one-dimensional viscous Burgers equation has a mean-field simplification with intriguing probabilistic structure. In this talk we will highlight some of this structure and indicate connections between stochastic explosion and uniqueness theory for the cascade and differential equations, respectively. This is based on joint work with Radu Dascaliuc, Nick Michalowski, and Enrique Thomann.

Ilya Zaliapin, University of Nevada, Reno
Thursday, 9:00 – 10:00 AM

Random Trees and Their Applications: Combinatorial Trees [tutorial]

This is the first (tutorial) part of the talk focused on self-similarity for trees (see Yevgeniy Kovchegov for the second part). Here we focus on combinatorial binary trees (trees with no edge lengths). We review the essential concepts of self-similar tree analysis, including Horton-Strahler indexing, Tokunaga indexing, tree pruning, prune-invariance, Horton and Tokunaga self-similarity. We also discuss a mapping between trees and continuous functions that allows one to rephrase the problems related to self-similar trees in the language of stochastic processes on the real line. We overview some well-known and new results on tree self-similarity and introduce a geometric branching process (GBP) that generates a range of self-similar trees. The analysis of GBP explains the importance of the Tokunaga self-similar trees, which are well-known in the physics literature since the 70s. This tutorial sets up the stage for more advanced results on metric trees presented by Yevgeniy Kovchegov in the second part of the talk.